Iterative Network Pricing for Ridesharing Platforms

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Decision, Risk, and Operations
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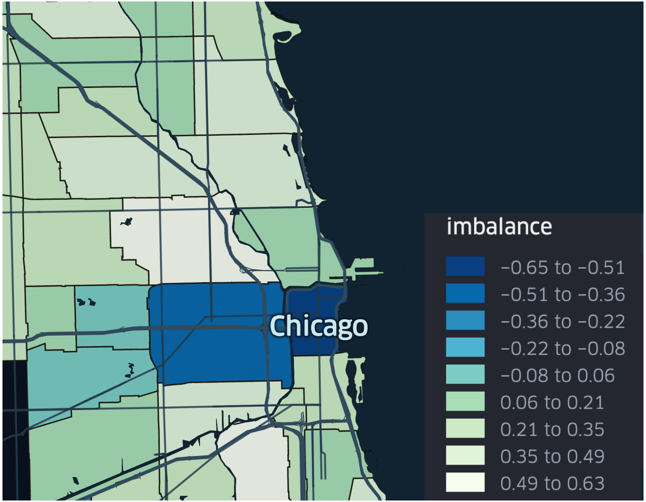
Chicago Trip Flow Imbalance: Morning Rush Hours



- Flow imbalance = (outflow − inflow) / (outflow + inflow)

https://data.cityofchicago.org/Transportation/Transportation-Network-Providers-Trips/m6dm-c72p

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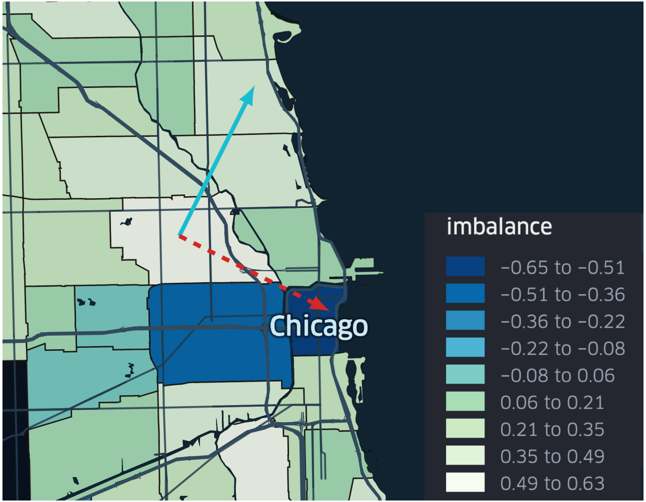

- Flow imbalance = (outflow − inflow) / (outflow + inflow)
- Origin-based pricing is inefficient

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Spatio-Temporal Pricing for Ridesharing Platforms

Optimal and incentive-compatible pricing:

$$(\text{trip price}) = (\text{trip cost}) + \underbrace{(\text{value of a driver at trip origin}) \\ - (\text{value of a driver at trip destination})}_{\text{network externality, needs full information}}$$

Hongyao Ma, Fei Fang, and David C. Parkes. Spatio-Temporal Pricing for Ridesharing Platforms. Operations Research, 2022.

Cashore J M, Frazier P I, Tardos E. Dynamic Pricing Provides Robust Equilibria in Stochastic Ride-Sharing Networks. ACM EC 2022: 301-302.

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Main challenge: lack of a demand model!

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What Might a Demand Function Look Like?

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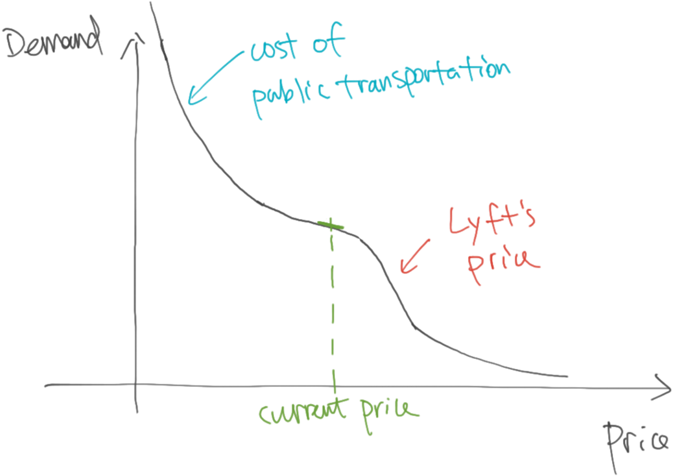

Related Work

- Theory: Banerjee, Johari, and Riquelme (2015), Castillo, Knoepfle, and Weyl (2017), Bimpikis, Candogan and Saban (2019), Ahmadinejad, Nazerzadeh, Saberi, Skochdopole, & Sweeney (2019), Yan, Zhu, Korolko and Woodard (2020), Ozkan and Ward (2020), Lian and van Ryzin (2021), Freund and van Ryzin (2021), Lian, Martin, and van Ryzin (2021), Besbes, Castro and Lobel (2021, 2022), Banerjee, Freund and Lykouris (2022), Garg and Nazerzadeh (2022), Afeche, Liu and Maglaras (2023), Asadpour, Lobel, & van Ryzin (2023), Kanoria and Qian (2023)

- Empirical: Chen and Sheldon (2015), Hall, Kendrick and Nosko (2015), Cohen, Hahn, Hall, Levitt, and Metcalfe (2016), Hall and Krueger (2016), Hall, Horton, and Knoepfle (2017), Lu, Frazier, and Kislev (2018), Chen, Rossi, Chevalier, and Oehlsen (2019), Xu, Vignon, Yin, and Ye (2019), Athey, Castillo and Chandar (2019), Castillo (2020), Cook, Diamon, List, Hall, and Oyer (2021), Angrist, Caldwell and Hall (2021), Rosaia (2023)

Price Trips to Downtown $1 Higher Than Lincoln Park?

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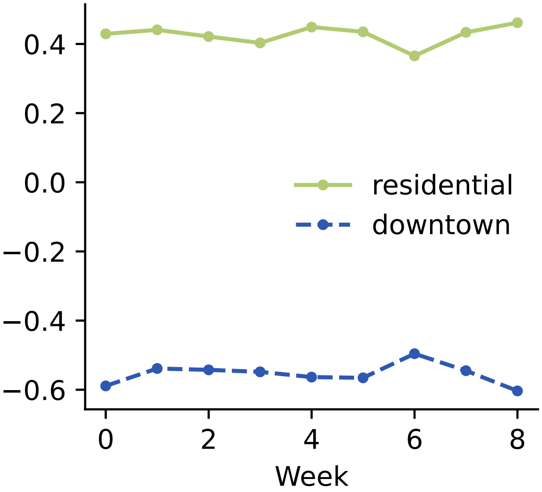

This Work: Iterative Network Pricing

Optimize origin-destination based prices via iterative adjustments, without using a rider demand model

Main Results (Informal)

- Maximum social welfare $\iff$ competitive equilibrium
- Suboptimality in social welfare $\leq O($difference in surge multipliers$)$
- Iterative Network Pricing $\rightarrow$ uniform surge multipliers $=$ maximum welfare

Outline

- Model
- Original-based market clearing
- Iterative Network Pricing
- Simulation results and discussions

Model

Stationary non-atomic supply and demand;

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- The platform can observe current demand level and local price elasticity

The Welfare-Optimal Outcome

- $x_{i,j}$ and $y_{i,j}$: rider and driver flows from i to j per unit of time, respectively

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- $v_{i,j}(\cdot)$: inverse function of $q_{i,j}(\cdot)$, i.e., $v_{i,j}(s)$ is the mininum value of the top $s \geq 0$ riders traveling from i to j (can cross zero)
- Maximize social welfare s.t. supply constraints and flow balance constraints

$$\underset{\boldsymbol{x},\boldsymbol{y} \in \mathbb{R}^{n^2}}{\text{maximize}} \quad \sum_{i,j\in\mathcal{L}} \left(\int_0^{x_{i,j}} v_{i,j}(s)\mathrm{d}s - c_{i,j}y_{i,j} \right) \tag{1a}$$

$$\text{subject to} \quad x_{i,j} \leq y_{i,j}, \qquad \forall i,j \in \mathcal{L}, \tag{1b}$$

$$\sum_{i,j\in\mathcal{L}} d_{i,j}y_{i,j} \leq m, \tag{1c}$$

$$\sum_{j\in\mathcal{L}} y_{i,j} = \sum_{j\in\mathcal{L}} y_{j,i}, \quad \forall i \in \mathcal{L}. \tag{1d}$$

Competitive Equilibrium

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- rider best-response: riders are picked-up iff. their value is above the price
- driver best-response: no driver can benefit from (i) choosing any alternative route and any subset of trips on the route, or (ii) not driving for the platform

A Welfare Theorem

Lemma (Welfare Theorem). A feasible rider and driver flow $(\boldsymbol{x}, \boldsymbol{y})$ is welfare-optimal if and only if there exists anonymous trip prices $\boldsymbol{p}$ that support the outcome in CE.

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Lemma (Price Structure). For any CE outcome $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{p})$, there exists $\omega^* \geq 0$ and $\boldsymbol{\phi}^* \in \mathbb{R}^n$ such that for all trips $(i, j) \in \mathcal{L}^2$,

$$p_{i,j} = c_{i,j} + d_{i,j}\omega^* + (\phi_i^* - \phi_j^*). \tag{2}$$

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- ω^*: the "surge multiplier", representing the marginal value of driver supply
- $\boldsymbol{\phi}^*$: the origin-destination (OD) based additive adjustments, corresponding to the marginal value of drivers at different locations

Duality and CE

Let $p_{i,j}$, ω, and ϕ_k be the dual variables corresponding to the feasibility constraints, (1b), total supply constraint (1c), and flow-balance constraints (1d), respectively.

$$\begin{aligned} \underset{\boldsymbol{p} \in \mathbb{R}^{n^2},\, \omega \in \mathbb{R},\, \boldsymbol{\phi} \in \mathbb{R}^n}{\text{minimize}} \quad & m\omega + \sum_{i,j \in \mathcal{L}} \int_0^{q_{i,j}(p_{i,j})} (v_{i,j}(s) - p_{i,j}) \mathrm{d}s && \text{(3a)} \\ \text{subject to} \quad & p_{i,j} = c_{i,j} + d_{i,j}\omega + \phi_i - \phi_j, \quad \forall i,j \in \mathcal{L}, && \text{(3b)} \\ & p_{i,j} \geq 0, \quad \forall i,j \in \mathcal{L}, && \text{(3c)} \\ & \omega \geq 0. && \text{(3d)} \end{aligned}$$

When prices are given by $\boldsymbol{p}$, ω can be interpreted as drivers' surplus rate, and the objective can be interpreted as the (nominal) total surplus of riders and drivers.

Optimality $\Leftrightarrow$ primal objective = dual objective $\Leftrightarrow$ driver and rider best-response.

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Caveat: Price cannot be negative. Once the price is too low, drivers will rather relocate without a rider.

(Coordinated) Origin-Based Market Clearing

Given $\phi \in \mathbb{R}^n$, and origin-based multipliers $\boldsymbol{\pi} \in \mathbb{R}^n$, prices are of the form

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Maximum slackness $e_{i,j} \triangleq \sup_{r\geq 0} r \cdot \tilde{q}_{i,j}(r)$

Existence, Uniqueness, and Optimality

Lemma (Existence and Uniqueness). Given any OD-based adjustments $\phi \in \mathbb{R}^n$, there exists a unique set of origin-based multipliers $\boldsymbol{\pi} \in \mathbb{R}^n$ that clears the market.

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Lemma (Optimality). The difference between the optimal welfare and the welfare achieved under an origin-based market-clearing outcome with multipliers $\boldsymbol{\pi}$ is at most:

$$m\left(\max_{i\in\mathcal{L}} \pi_i - \min_{i\in\mathcal{L}} \pi_i\right) + m\left(0 - \max_{i\in\mathcal{L}} \pi_i\right)^{+} + \sum_{i,j\in\mathcal{L}} e_{i,j}. \quad (5)$$

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Suggests minimizing $\max_i \pi_i - \min_i \pi_i$.

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Denote $\boldsymbol{\Pi}$ as the mapping from $\boldsymbol{\phi}$ to the market-clearing multipliers $\boldsymbol{\pi}$.

Lemma. $\boldsymbol{\Pi}$ is continuously differentiable, and the Jacobian matrix $\mathrm{D}\boldsymbol{\Pi}$ at any $\boldsymbol{\phi}$ can be computed using the corresponding market-clearing outcome.

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Denote Π as the mapping from ϕ to the market-clearing multipliers π.

Lemma. Π is continuously differentiable, and the Jacobian matrix $\mathrm{D}\Pi$ at any ϕ can be computed using the corresponding market-clearing outcome.

After replacing π with $\Pi(\phi)$, the dual objective as a function of ϕ is not convex!

Iterative Network Pricing (Intuition)

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- A damped Newton's method with backtracking line search

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At every $t > 0$, after observing the market-clearing outcome of period $t - 1$:

Iterative Network Pricing

At $t = 0$: start from the initial market-clearing outcome with $\phi^{(0)} = \mathbf{0}$.

At every $t > 0$, after observing the market-clearing outcome of period $t-1$:

- move $\phi^{(t)}$ in the direction of $\left[-\mathrm{D}\boldsymbol{\Pi}(\phi^{(t-1)}) \quad \mathbf{1}\right]^{-1} \boldsymbol{\pi}^{(t-1)}$, for a step-size such that no π_i is expected to change by more than τ,

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- if there is not sufficient improvement in f, where $f(\phi) \triangleq \sum_{i \in \mathcal{L}} \left(\Pi_i(\phi) - \frac{1}{n} \sum_{j \in \mathcal{L}} \Pi_j(\phi)\right)^2$, reduce the size of the previous step.

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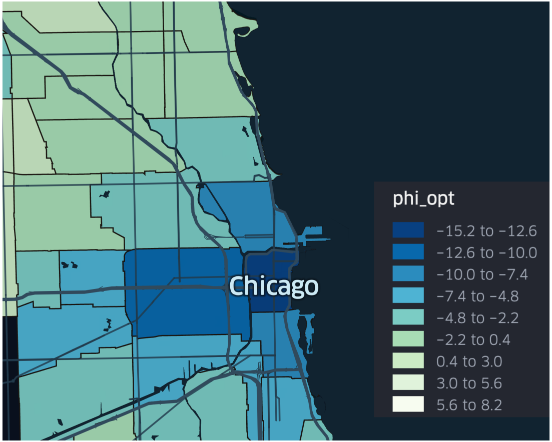
Theorem

The INP mechanism converges to an outcome where all origin-based multipliers are equal, i.e. $\exists\omega^\star$ s.t. $\lim_{t\to\infty}\pi_i^{(t)} = \omega^\star$ for all $i \in \mathcal{L}$. The welfare-suboptimality of the limiting outcome is upper bounded by $m\max\{0, -\omega^\star\} + \sum_{i,j\in\mathcal{L}} e_{i,j}$.

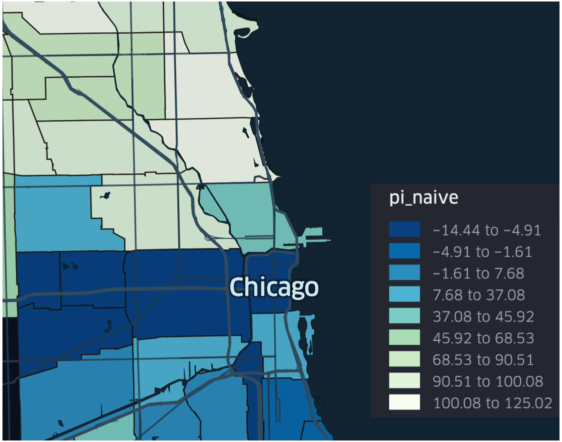
Outline

- Model
- Original-based market clearing
- Iterative Network Pricing
- Simulation results and discussions

Chicago Morning Rush Hours

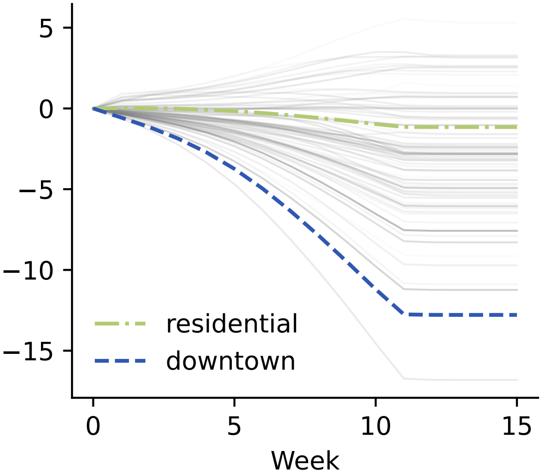


(a) Optimal OD-based adjustments

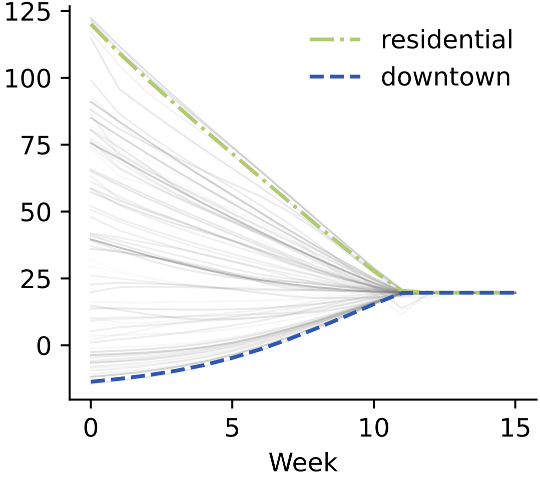


(b) The naive origin-based multipliers

Stationary Market Condition (1 of 3)

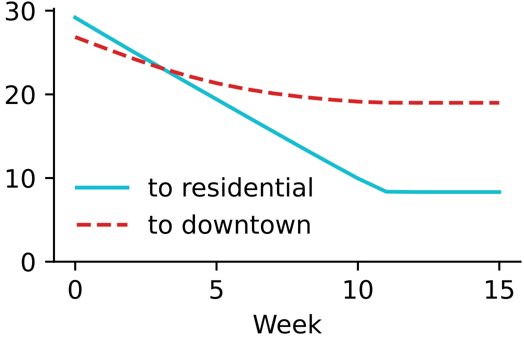


(a) OD-based adjustments $\phi^{(t)}$

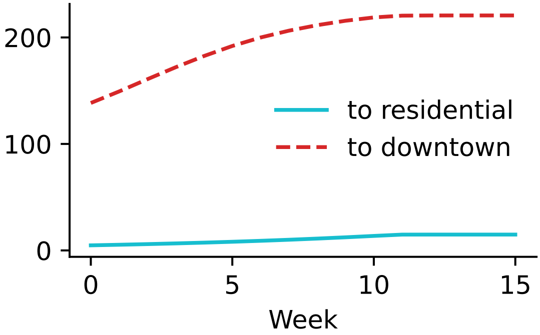


(b) Origin-based multipliers $\pi^{(t)}$

Stationary Market Condition (2 of 3)

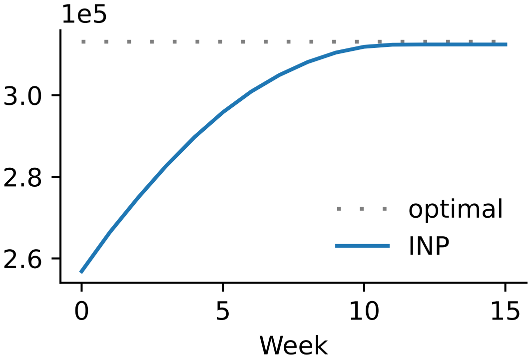


(a) Trip prices

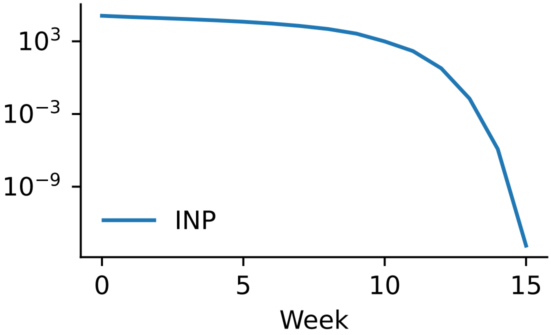


(b) Rider demand

Stationary Market Condition (3 of 3)

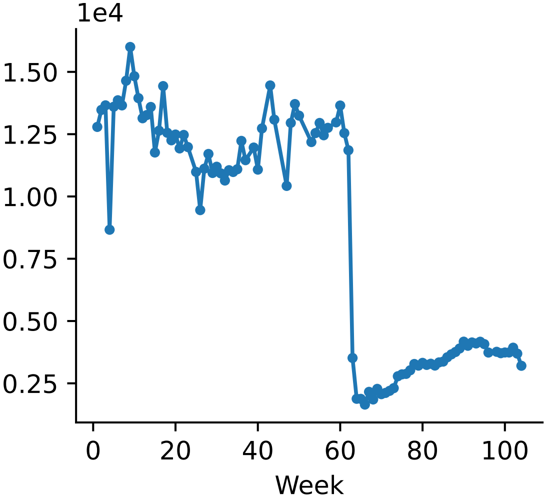


(a) Social welfare

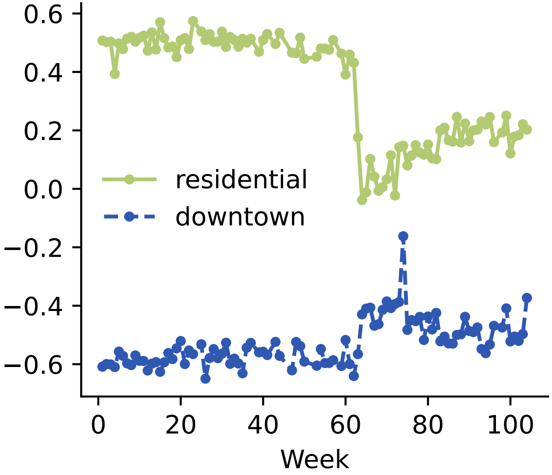


(b) The Lyapunov function f

Non-stationary Market Condition: Wed 8-9am 2019-2020

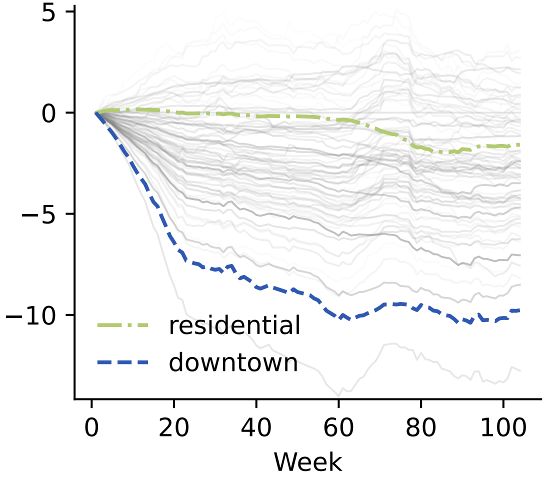


(a) Total trip volume

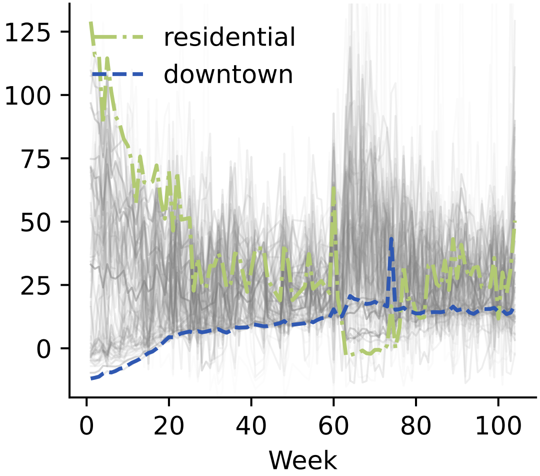


(b) Flow imbalance

$\phi^{(t)}$ and $\pi^{(t)}$ Under INP

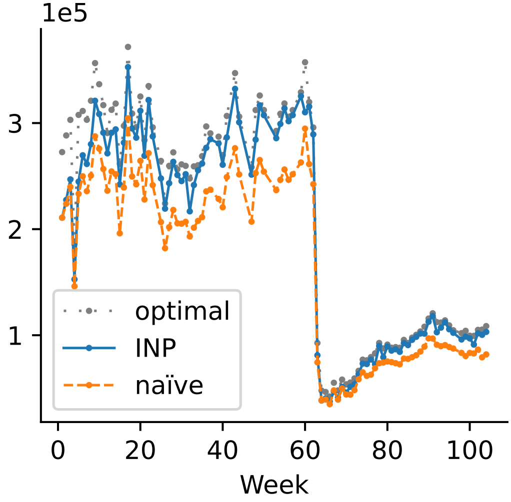


(a) OD-based adjustments $\phi^{(t)}$

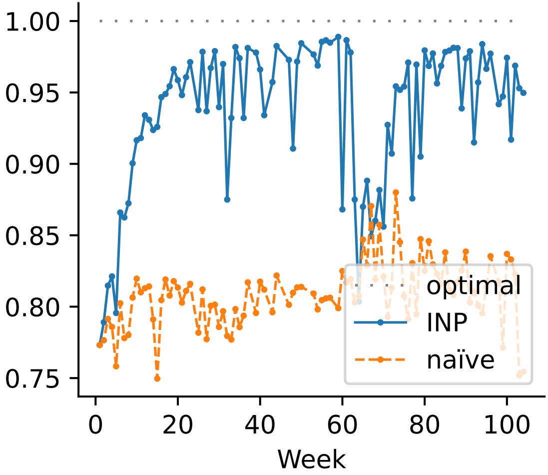


(b) Origin-based multipliers $\pi^{(t)}$

Social Welfare and Welfare Ratio

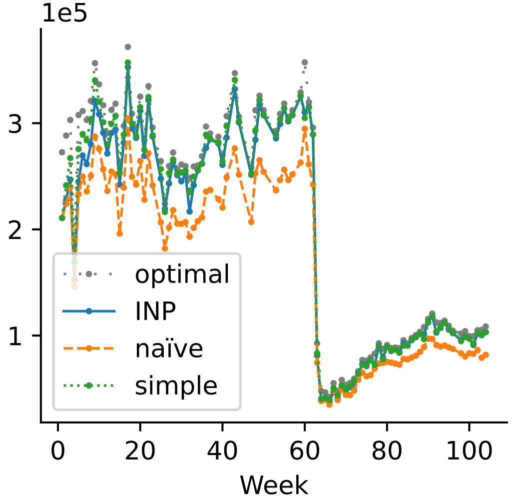


(a) Social welfare

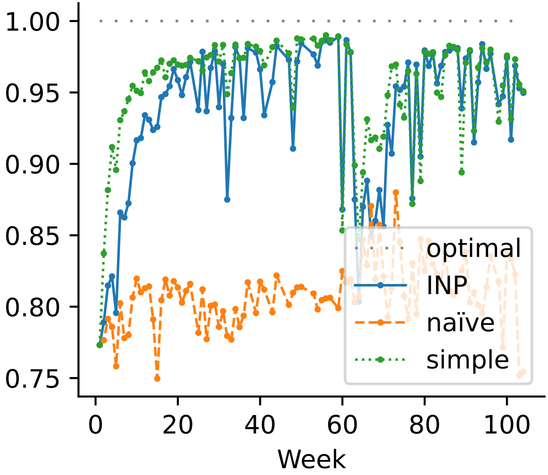


(b) Fraction of optimal welfare

Origin-Based Multipliers $\pi^{(t-1)}$ as the Update Direction



(a) Social welfare



(b) Fraction of optimal welfare

Discussions

This work: optimal origin-destination based pricing without a demand model

Discussions

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Related literature on iterative mechanisms, e.g. combinatorial auctions

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Next steps

- Non-stationary market conditions and slow market equilibration

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- Noisy observations of rider demand, elasticity, and driver earnings

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Related literature on iterative mechanisms, e.g. combinatorial auctions

Next steps

- Non-stationary market conditions and slow market equilibration
- Noisy observations of rider demand, elasticity, and driver earnings
- Strategic drivers, and competition between platforms

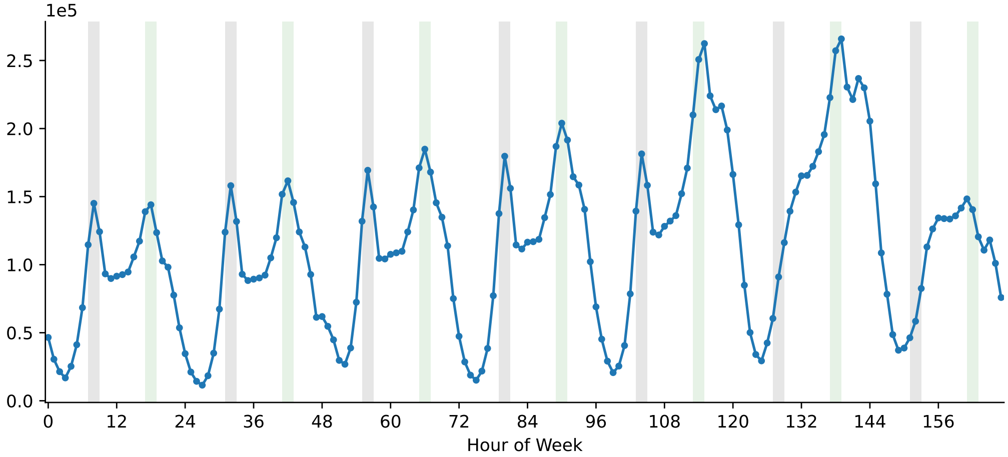
Thank You!

https://arxiv.org/abs/2311.08392

APPENDIX

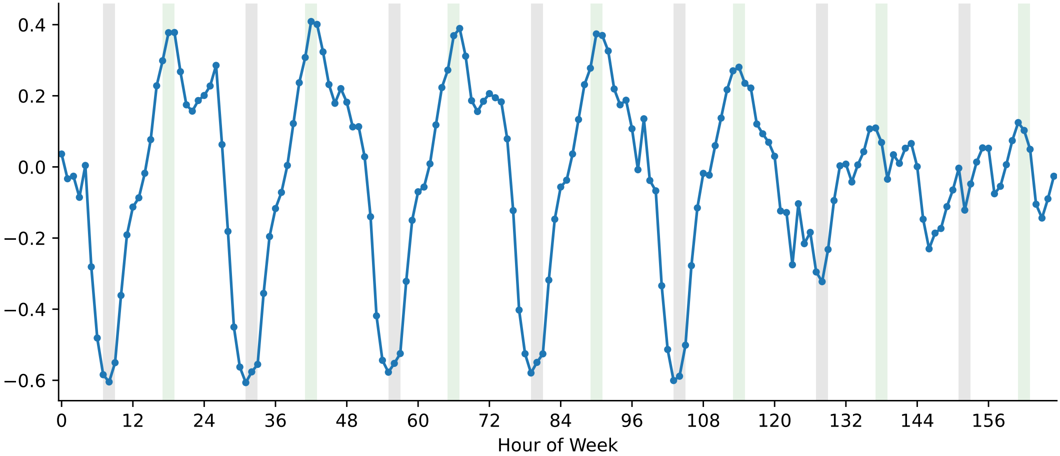
Chicago Market Dynamics

Trip Volume by Hour-of-Week



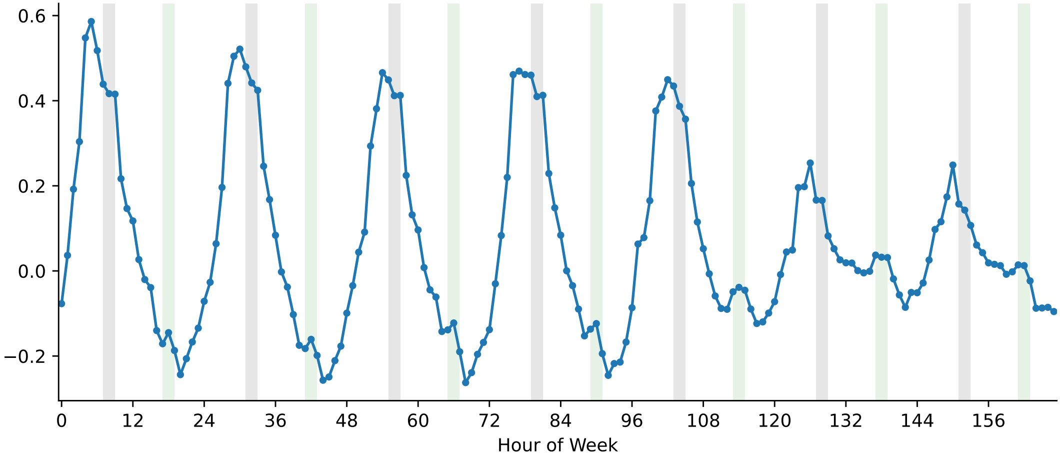
Gray bar: morning rush hours 7-9am; Green bar: evening rush hours 5-7pm

Flow Imbalance by Hour-of-Week: The Loop



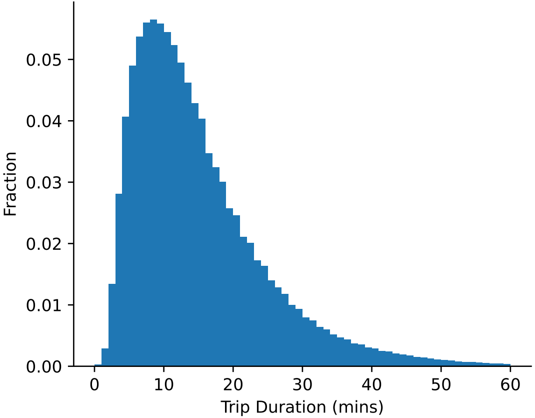
Gray bar: morning rush hours 7-9am; Green bar: evening rush hours 5-7pm

Flow Imbalance by Hour-of-Week: Lake View

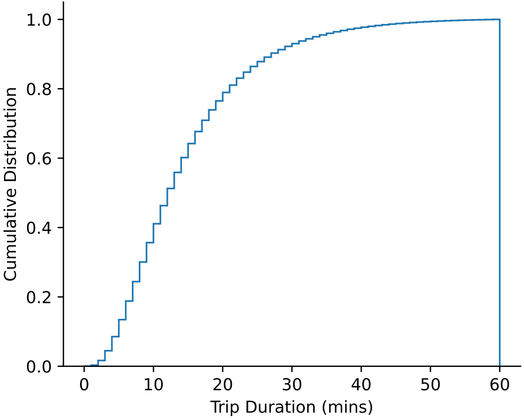


Gray bar: morning rush hours 7-9am; Green bar: evening rush hours 5-7pm

Distribution of Trip Duration



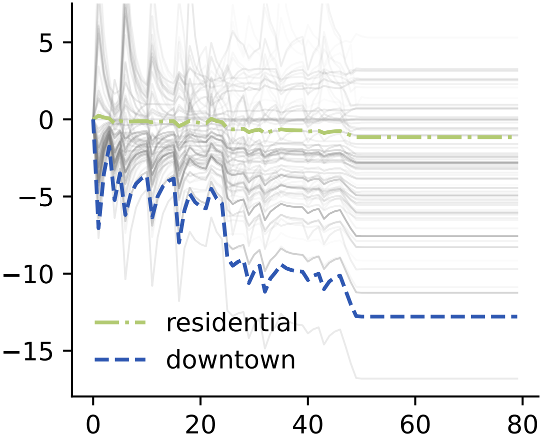
(a) Distribution of trip duration



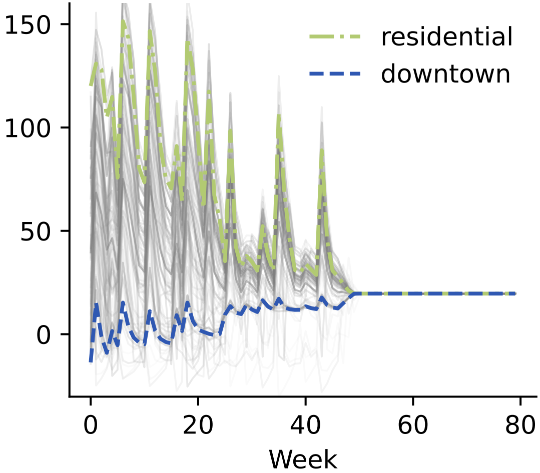
(b) Cumulative distribution of trip duration

Additional Simulations: Stationary Market Conditions

The “Pure Newton” Method

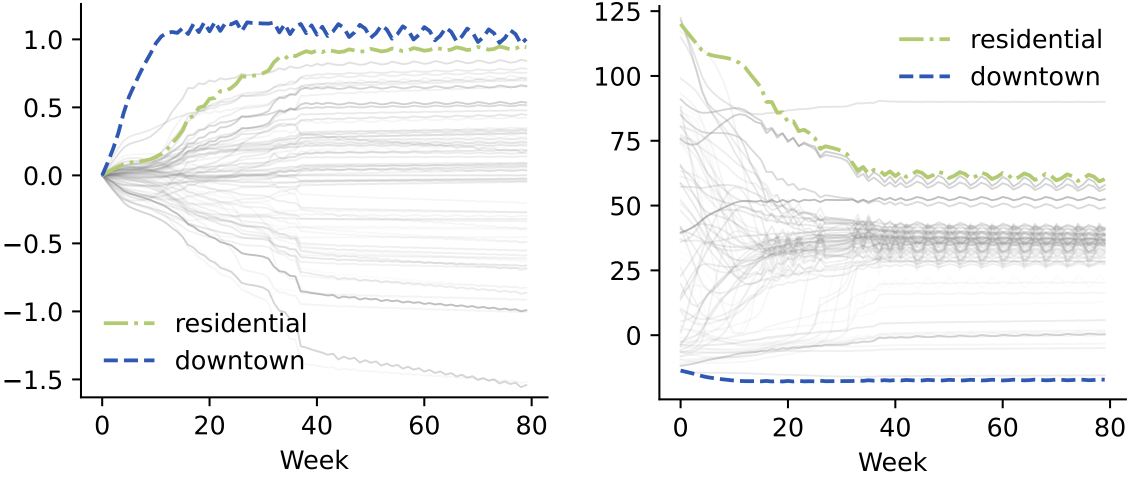


(a) OD-based adjustments $\phi^{(t)}$



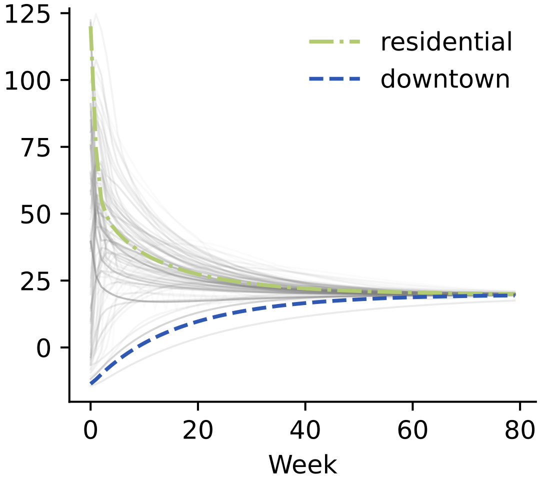
(b) Origin-based multipliers $\pi^{(t)}$

Gradient Descent w.r.t. f

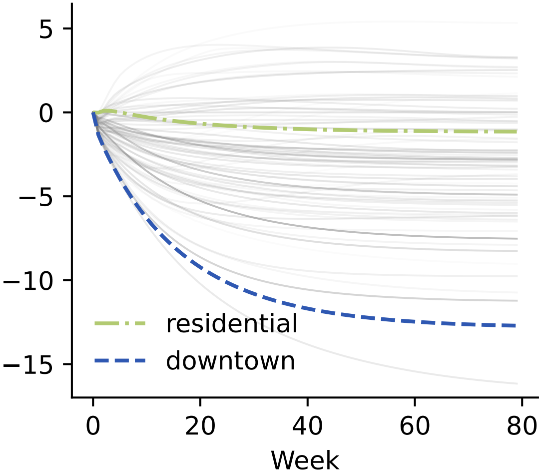


(a) OD-based adjustments $\phi^{(t)}$ (b) Origin-based multipliers $\pi^{(t)}$

π as the Direction (1 of 3)

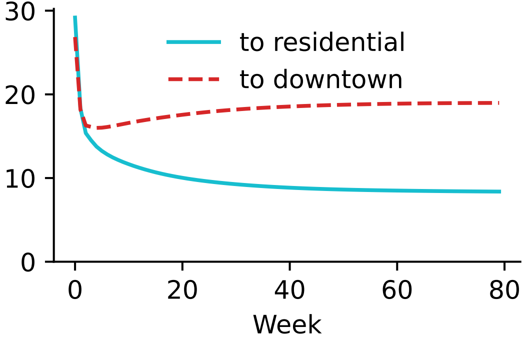


(a) OD-based adjustments $\phi^{(t)}$

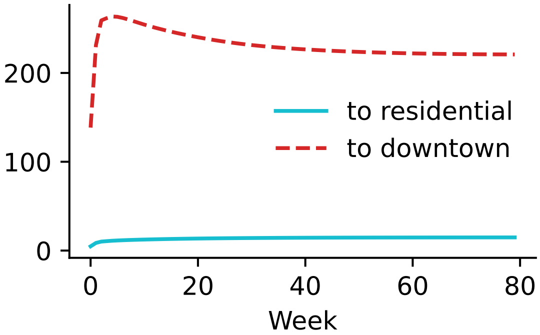


(b) Origin-based multipliers $\pi^{(t)}$

Chicago Morning Rush: π as the Direction (2 of 3)

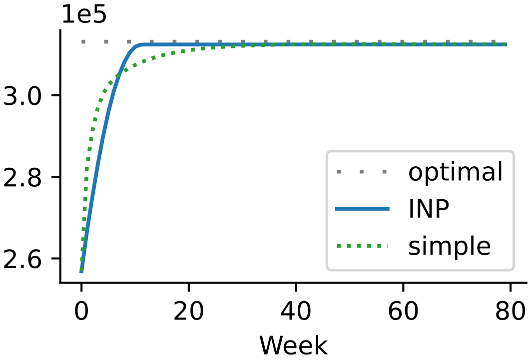


(a) Trip prices

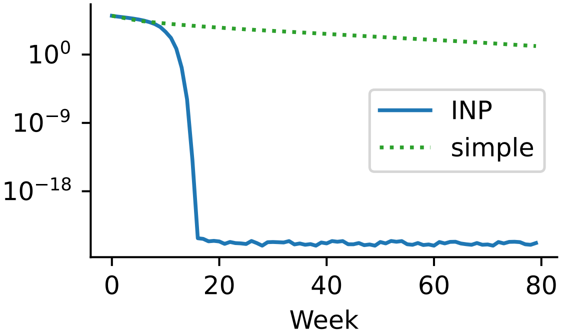


(b) Rider demand

π as the Direction (3 of 3)



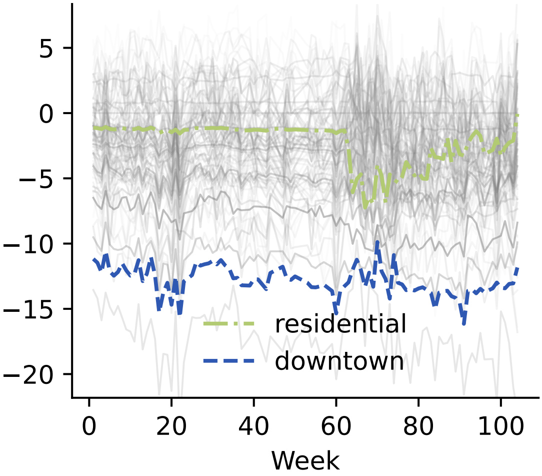
(a) Social welfare



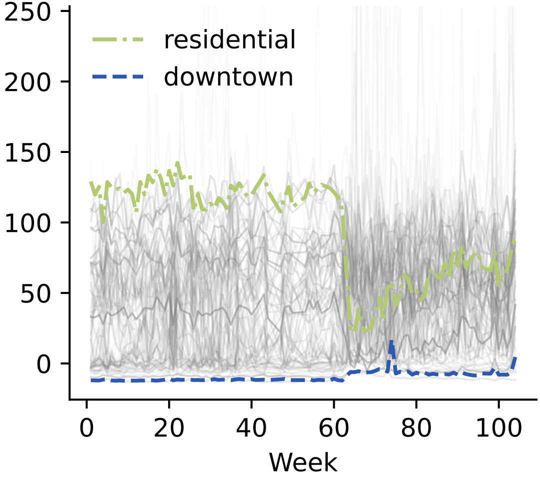
(b) The Lyapunov function f

Additional Simulations: Chicago 2019-2020

Optimal ϕ and naive π

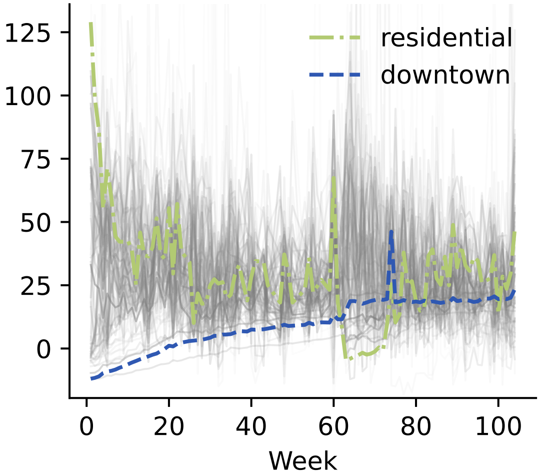


(a) Optimal OD-based adjustments $\phi^{(t)}$

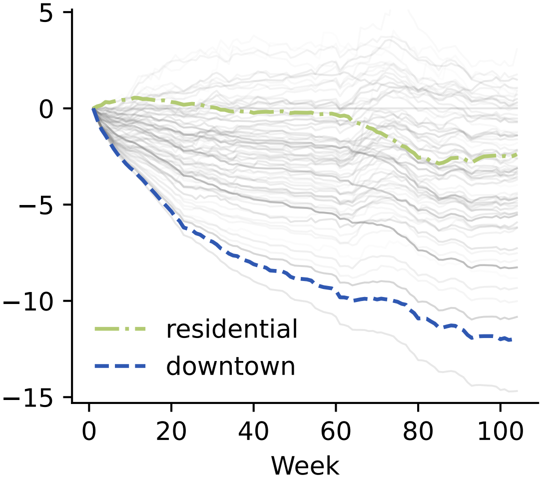


(b) Naive origin-based multipliers $\pi^{(t)}$

π as the Direction



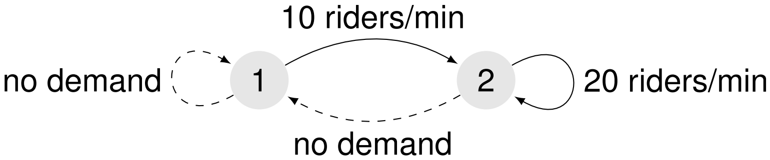
(a) OD-based adjustments $\phi^{(t)}$



(b) Origin-based multipliers $\pi^{(t)}$

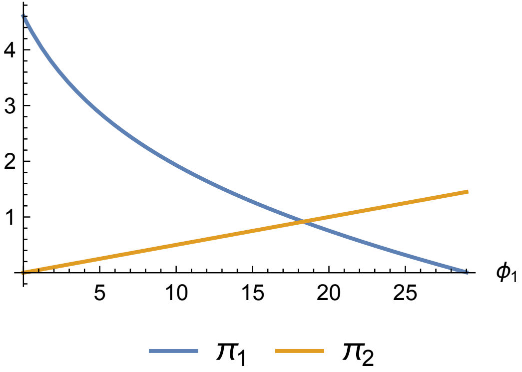
A Two-Location Example: Morning Rush Hour

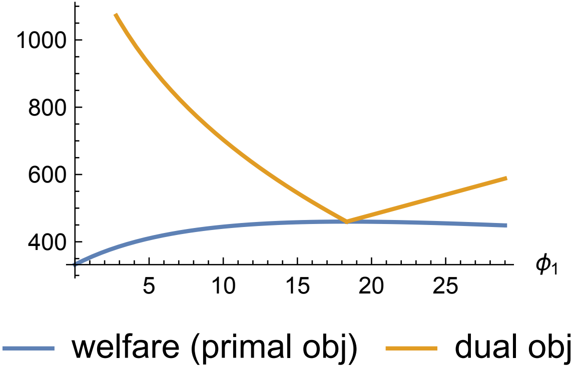
Example: Morning Rush Hour



- Trips within each location takes 10 mins to complete while trips between locations take 20 mins.
- Rider values are exponentially distributed, with mean values 40 for the (1,2) trip, and 10 for the (2,2) trip.
- A total of $m = 240$ drivers are available, and assume zero trip costs.
- Under the welfare-optimal outcome, $y_{1,2} = y_{2,1} = 4$ and $y_{2,2} = 8$. Moreover, $\omega \approx 0.916$, $\phi_1 - \phi_2 \approx 18.33$, leading to $p_{1,2} = d_{1,2}\omega + \phi_1 - \phi_2 = 4p_{2,2} \approx 36.64$.
- The naive "origin-based market-clearing" outcome has $\pi_2 = 0$ and $\pi_1 > 4$.

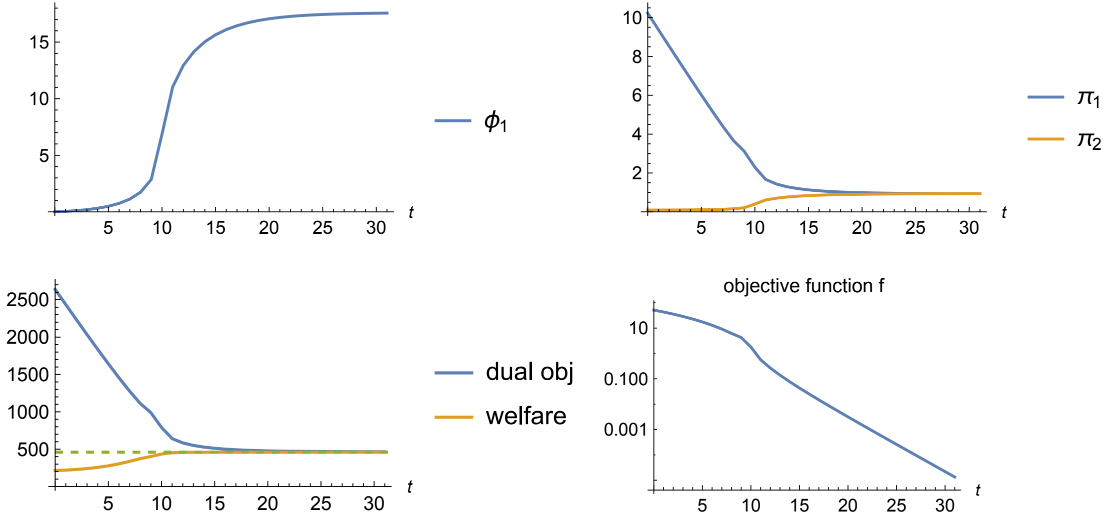
Morning Rush Hour: Origin-Based Market Clearing



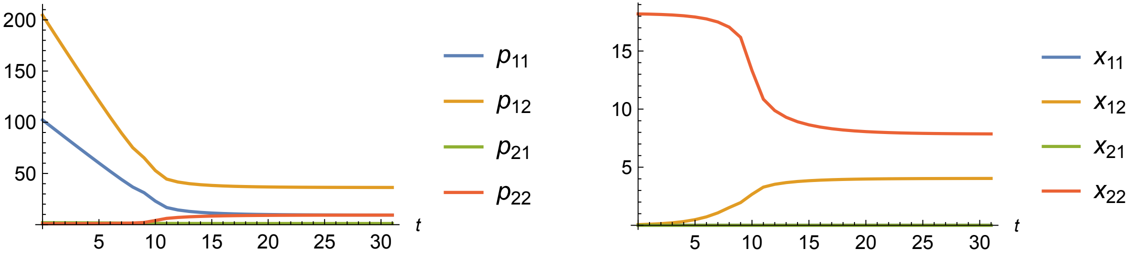


Assumption: undispatched drivers at location 2 relocate back to location 1.

Morning Rush Hour: Iterative Network Pricing



Morning Rush Hour: Iterative Network Pricing (Cont)



Reliable yet Flexible

Reliability for riders:

"Transportation as reliable as running water, everywhere, for everyone"

Flexibility for drivers:

"Work that puts you first— drive when you want, earn what you need"

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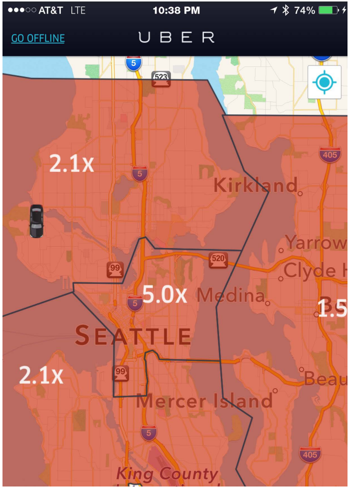
http://time.com/time-person-of-the-year-2015-runner-up-travis-kalanick/

https://www.uber.com/drive/

Chen M K. Dynamic Pricing in a Labor Market: Surge Pricing and Flexible Work on the Uber Platform. ACM EC 2016: 455-455.

Market Failure (1 of 3): Prices Not Spatially Smooth

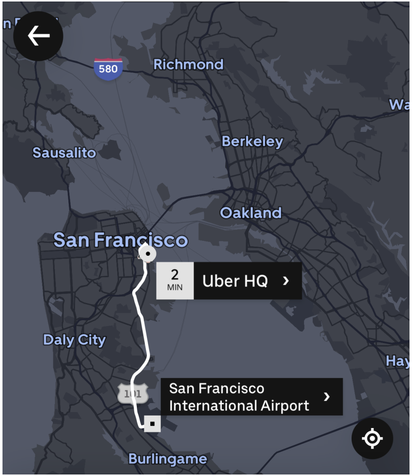


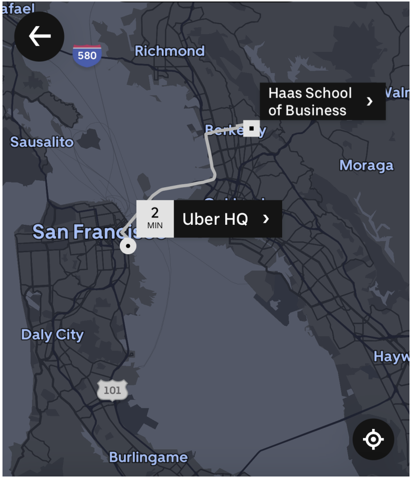
https://uberpeople.net/threads/for-the-seattle-veterans.223372/

Market Failure (2 of 3): Prices Not Temporally Smooth

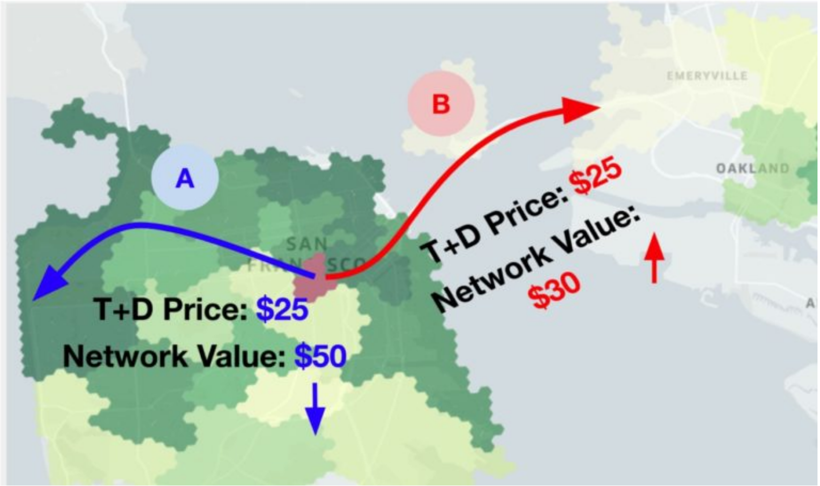
http://www.nbcsports.com/boston/boston-bruins/five-games-to-watch-on-boston-bruins-schedule

Market Failure (3 of 3): Destinatio-Oblivious Surge



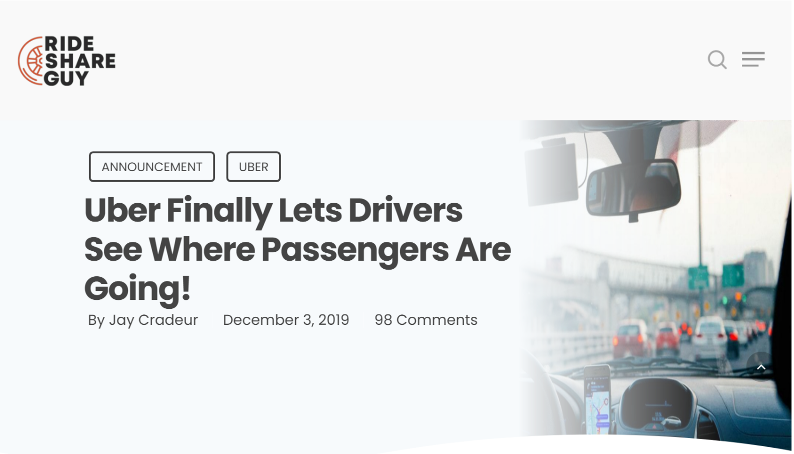


“Network Value” by Destination



https://web.archive.org/web/20210825043610/https://eng.uber.com/powering-network-pricing-model/

Uber's Upfront Trip Information



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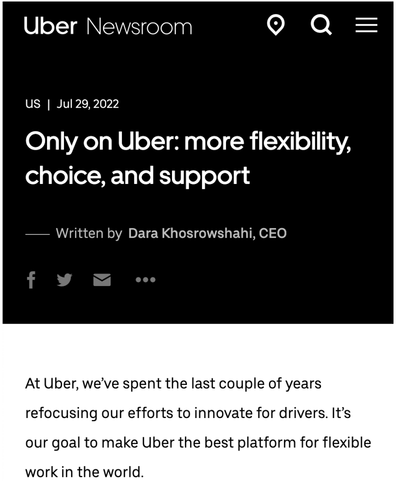
https://therideshareguy.com/uber-rolling-out-new-driver-features/

https://www.uber.com/blog/california/keeping-you-in-the-drivers-seat-1/

Driver Upfront Fare and Destination on Uber

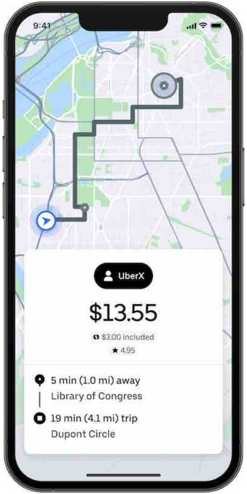

Uber Newsroom

US | Jul 29, 2022

Only on Uber: more flexibility, choice, and support

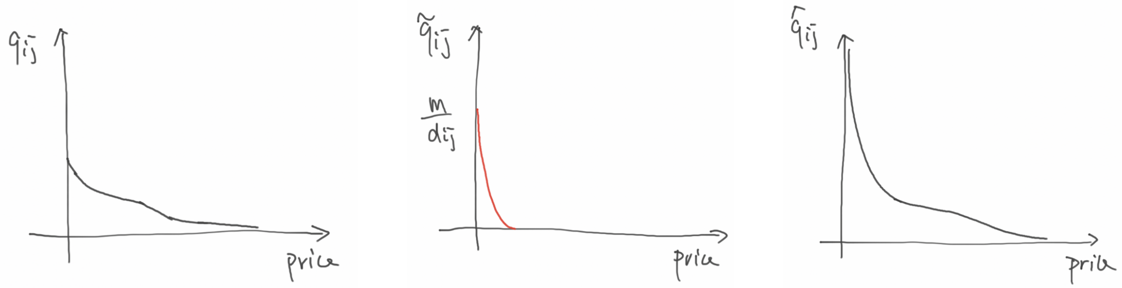
Written by **Dara Khosrowshahi, CEO**

At Uber, we've spent the last couple of years refocusing our efforts to innovate for drivers. It's our goal to make Uber the best platform for flexible work in the world.



https://www.uber.com/newsroom/only-on-uber/

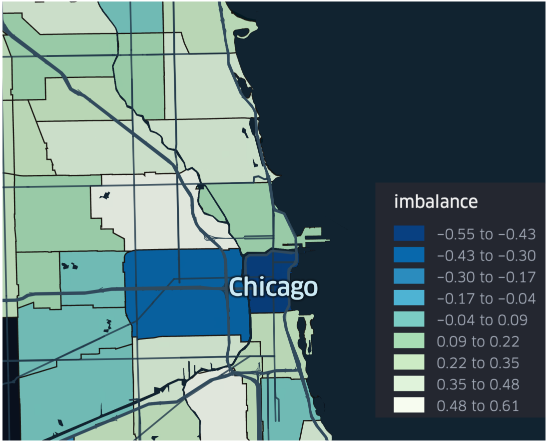
“Phantom Demand” Coordinates Driver Relocation



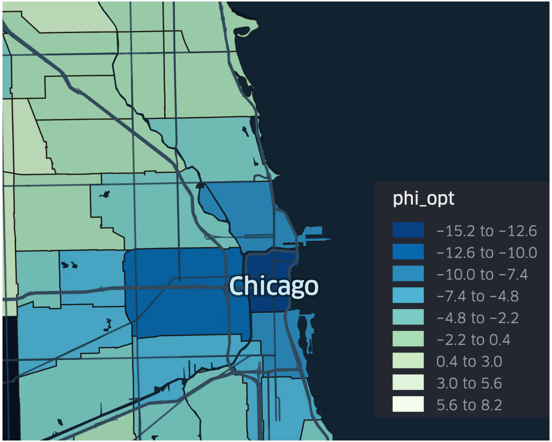
rider demand + phantom demand = effective total demand

- For each trip $(i, j) \in \mathcal{L}^2$, construct $\tilde{q}_{i,j}$ s.t. $\tilde{q}_{i,j}(0) = m/d_{i,j}$
- The maximum “slackness” $e_{i,j} \triangleq \sup_{r \geq 0} r \cdot \tilde{q}_{i,j}(r)$ should not be too big

Chicago Morning Rush Hours: the Optimal ϕ^*



(a) Status quo trip flow imbalance

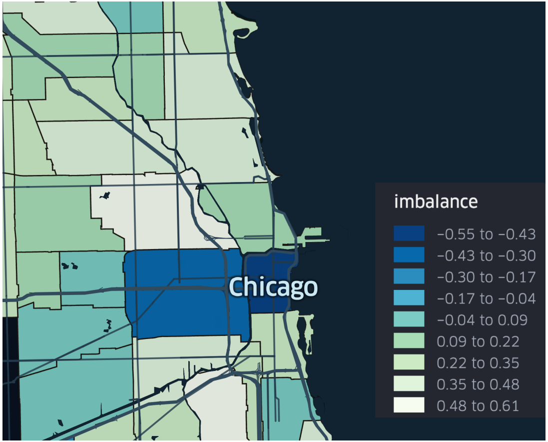


(b) The welfare-optimal ϕ^*

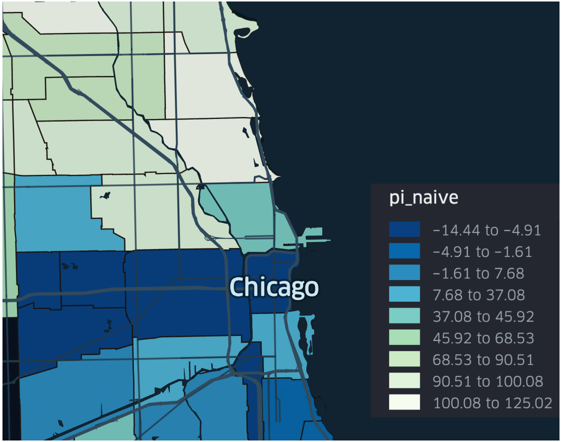
https://kepler.gl/demo/map?mapUrl=https://dl.dropboxusercontent.com/s/a3o2i8bfd6ra36a/keplergl_z4qmat6.json

Assumptions: rider values are exponentially distributed with mean \$1/min $\times$ trip duration; driver cost \$1/3/min. Min. number of drivers to fulfill the rider trips.

Chicago Morning Rush Hours: Origin-Based Surge



(a) Status quo trip flow imbalance



(b) Naive surge multipliers π
(i.e. $p_{i,j} = c_{i,j} + d_{i,j}\pi_i$)

Assumptions: rider values are exponentially distributed with mean \$1/min $\times$ trip duration; driver cost \$1/3/min. Min. number of drivers to fulfill the rider trips.

The Primal (with Redundant Constraints)

$$\begin{aligned} \underset{\boldsymbol{x},\boldsymbol{y}\in\mathbb{R}^{n^2},\,\boldsymbol{z}\in\mathbb{R}^n}{\text{maximize}} \quad & \sum_{i,j\in\mathcal{L}}\left(\int_0^{x_{i,j}} v_{i,j}(s)\mathrm{d}s - c_{i,j}y_{i,j}\right) && \text{(6a)}\\ \text{subject to} \quad & x_{i,j}\le y_{i,j}, \qquad \forall i,j\in\mathcal{L}, && \text{(6b)}\\ & \sum_{i,j\in\mathcal{L}} d_{i,j}y_{i,j} = z_i, \qquad \forall i\in\mathcal{L}, && \text{(6c)}\\ & \sum_{i\in\mathcal{L}} z_i \le m, && \text{(6d)}\\ & \sum_{j\in\mathcal{L}} y_{k,j} = \sum_{i\in\mathcal{L}} y_{i,k}, \quad \forall k\in\mathcal{L}. && \text{(6e)} \end{aligned}$$

(6c) and (6d) replace the original total supply constraint $\sum_{i,j\in\mathcal{L}} d_{i,j}y_{i,j}\le m$.

The Dual (with Origin-Based Multipliers)

$$
\begin{aligned}
\underset{\boldsymbol{p} \in \mathbb{R}^{n^2},\, \omega \in \mathbb{R},\, \boldsymbol{\pi} \in \mathbb{R}^n,\, \boldsymbol{\phi} \in \mathbb{R}^n}{\text{minimize}} \quad & m\omega + \sum_{i,j \in \mathcal{L}} \int_0^{q_{i,j}(p_{i,j})} (v_{i,j}(s) - p_{i,j}) \mathrm{d}s && \text{(7a)} \\
\text{subject to} \quad & p_{i,j} = c_{i,j} + d_{i,j}\pi_i + \phi_i - \phi_j, \quad \forall i,j \in \mathcal{L}, && \text{(7b)} \\
& p_{i,j} \geq 0, \quad \forall i,j \in \mathcal{L}, && \text{(7c)} \\
& \omega \geq \pi_i, \quad \forall i \in \mathcal{L}, && \text{(7d)} \\
& \omega \geq 0. && \text{(7e)}
\end{aligned}
$$

Dual Obj − Primal Obj = Violation of Best Response

$$\sum_{i,j\in\mathcal{L}} \int_{x_{i,j}}^{q_{i,j}(p_{i,j})} (v_{i,j}(s) - p_{i,j})\mathrm{d}s \tag{8a}$$

$$+\sum_{i\in\mathcal{L}} z_i(\omega - \pi_i) \tag{8b}$$

$$+\sum_{i,j\in\mathcal{L}} p_{i,j}(y_{i,j} - x_{i,j}) \tag{8c}$$

$$+\omega\left(m - \sum_{i\in\mathcal{L}} z_i\right). \tag{8d}$$

- (8a): violation of “riders are picked up iff. their value is aboce the price”
- (8b): violation of “all drivers getting highest possible surplus rate”
- (8c): violation of “price must be zero if supply exceeds demand”
- (8d): violation of “no driver is sent home if some driver gets positive surplus”